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Conformal Killing's Equations of Einstein Closed Static Universe Metric in Different Geometries

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Abstract: **The aim of this paper is to find and solve the conformal killing's equations of Einstein Closed Static Universe metric in Lyra geometry and Riemannian geometry, obtained conformal killing vector fields and compare between the results.**

Keywords: **Christoffal symbols, Conformal Killing vector field, homothetic, Lyra geometry, Riemannian Geometry, Static Universe metric.**

I. INTRODUCTION

A static universe is a cosmological model in which the universe is both spatially infinite and temporally infinite, and space is neither expanding nor contracting. Such a universe does not have spatial curvature; that is to say that it is 'flat' or Euclidean. A static infinite universe was first proposed by Thomas Digges. In contrast to this model, Albert Einstein proposed a temporally infinite but spatially finite model as his preferred cosmology in 1917, in his paper Cosmological Considerations in the General Theory of Relativity.[1],[2]. where the metric g_{ij} is symmetric of rank four it has ten independent components thus we get ten conformal killing's equations which solved to get the conformal killing vector field. Lyra geometry has proposed a modification of Riemannian geometry by introducing a gauge function into the structure less manifold that bears a close resemblance to Weyl's geometry. The paper is organized as follows: In section II we summarize some of the basic concepts of Lyra Geometry, which will be used though this work, Section III deals with the model and evaluating solution of the conformal , homothetic and killing' s equations in Riemannian geometry . In section IV evaluating the solution of conformal killing's equations , homothetic and killing's equations in Lyra geometry [3]-[6]

Problem statement and objectives:

Where studying some of the models of metric space times in the Lyra geometry it is difficult to get conformal, homothetic and killing equations as well as to get solved. In this paper we calculate the equations in the ordinary method as well as using a computer program and compare the results to be able to use computer programs in difficult models.

Methods:

In this paper we get conformal killing , homothetic and killing's equations by using equation(II.5),where$\psi(t,r,\vartheta,\varphi)$, ψ is constante and zero respectively we solve the partial differential equations by separate variables and use Maple 17 program for getting the solution of the ten equations in each case. [7]

II. THE VERSION OF MODEL AND CONFORMAL VECTOR FIELD IN LYRA'S GEOMETRY

An n-dimensional Lyra manifold M is a generalization to the Riemannian manifold For any point $p \in M$ we can define the coordinate system $\{x^{\mu}\}_{\mu=1}^{n}$. In addition to these coordinate there exist a gauge function $x^{o} = x^{o}(x^{\mu})_{\mu=1}^{n}$ which together with $\{x^{\mu}\}_{\mu=1}^{n}$ form a reference system transformation $(x^{o}, x^{\mu})_{\mu=1}^{n}$. In Lyra geometry the metric or the measure of length of displacement vector $\zeta^{\mu} = x^{o}dx^{\mu}$ between two points $p(x^{\mu})_{\mu=1}^{n}$ and $q(x^{\mu} + dx^{\mu})_{\mu=1}^{n}$ is given by absolute invariant under both gauge function and coordinate system written as: [8]

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$$ds^2 = g_{\mu\nu} x^o \, dx^\mu dx^o dx^\nu \qquad \text{(II.1)}$$

Where $g_{\mu\nu}$ is a metric tensor as in Riemannian connection $\left\{ {\alpha \atop \mu\nu} \right\}$, but also by a function ϕ_μ, which arises through gauge transformation and it is given by

$$\left\{ {\alpha \atop \mu\nu} \right\} = \frac{1}{2} g^{\alpha\rho}(g_{\rho\nu,\mu} + g_{\rho\mu,\nu} - g_{\mu\nu,\rho}) \qquad \text{(II.2)}$$

$$\Gamma^\alpha_{\mu\nu} = (x^o)^{-1} \left\{ {\alpha \atop \mu\nu} \right\} + \frac{1}{2}\left(\delta^\alpha_\mu \phi_\nu + \delta^\alpha_\nu \phi_\mu - g_{\mu\nu}\phi^\alpha\right) \qquad \text{(II.3)}$$

Where ϕ is called a displacement vector field and satisfies $\phi^\alpha = g^{\alpha\gamma}\phi_\gamma$, we consider ϕ to be a timelike vector where

$$\phi^\alpha = g^{\alpha\gamma}\phi_\gamma \,, \quad \phi_\gamma = (\beta(t), 0,0,0) \qquad \text{(II.4)}$$

Throughout the paper M will denote a 4-dimensional Lyra manifold with Lorentz metric g which is a generalization to the 4-dimensional Riemannian manifold [9]

As in Riemannian geometry , a global vector field $\zeta = \zeta^\mu(t, r, \vartheta, \varphi)^3_{\mu=0}$ on M is called conformal vector field if the following condition holds:

$$\mathcal{L}_\eta g_{\mu\nu} = g_{\rho\nu}\nabla_\mu \eta^\rho + g_{\mu\rho}\nabla_\nu \eta^\rho = 2\psi g_{\mu\nu} \;,\; \psi(\, t, r, \vartheta, \varphi\,) \qquad \text{(II.5)}$$

where ψ is constant we get the homothetic equations, where $\psi = 0$ we get the killing equations, $\mathcal{L}$ denote a Lie derivatives and ∇ is the covariant derivative such that:

$$\left.\begin{aligned} \nabla_\mu \zeta^\rho &= \frac{1}{x^o}\partial_\mu \zeta^\rho + \Gamma^\rho_{\mu\alpha}\zeta^\alpha \\ \nabla_\mu \zeta_\rho &= \frac{1}{x^o}\partial_\mu \zeta_\rho - \Gamma^\alpha_{\mu\rho}\zeta_\alpha \end{aligned}\right\} \qquad \text{(II. 6)}$$

Where $\Gamma^\alpha_{\mu\nu}$ is a Lyra connection form given by (II.2) in equation (II.3).

III. CONFORMAL KILLING'S EQUATIONS AND THEIR SOLUTION IN RIEMANNIAN GEOMETRY

$$ds^2 = dt^2 - dr^2 + sin^2 r(d\vartheta^2 + \sin^2\vartheta d\varphi^2) \qquad \text{(III. 1)}$$

Where $x^0 = t, x^1 = r, x^2 = \vartheta, x^4 = \varphi$

We get the nine non vanishing Christoffel symbols of second kind in Riemannian geometry from (II.2) as :

$$\left\{ {r \atop \vartheta\vartheta} \right\} = -sinr\ cosr \;,\; \left\{ {r \atop \varphi\varphi} \right\} = -sinr\ cosr\ \sin^2\vartheta \;,\; \left\{ {\varphi \atop \vartheta\varphi} \right\} = \left\{ {\varphi \atop \varphi\vartheta} \right\} = \frac{cos\vartheta}{sin\vartheta},$$

$$\left\{ {\vartheta \atop \varphi\varphi} \right\} = -sin\vartheta\ cos\vartheta \;,\; \left\{ {\vartheta \atop r\vartheta} \right\} = \left\{ {\vartheta \atop \vartheta r} \right\} = \left\{ {\varphi \atop r\varphi} \right\} = \left\{ {\varphi \atop \varphi r} \right\} = \frac{cosr}{sinr} \qquad \text{(III.2)}$$

We use equation (II.5) to deduce the following system of the ten Conformal killing's equations:[10]

$$\frac{1}{2} sin2\vartheta\zeta^2 + \frac{1}{2}\, sin^2\vartheta sin2r\ \zeta^1 + \zeta^3_{,\varphi} = -2sin^2 r\ sin^2\vartheta\ \Omega(t,r,\vartheta,\varphi) \qquad \text{(III.3)}$$

$$\zeta^2_{,\vartheta} + \frac{1}{2} sin2r\ \zeta^1 = -2\sin^2 r\ \Omega(t,r,\vartheta,\varphi) \qquad \text{(III.4)}$$

$$\zeta^1_{,r} = -2\Omega(t,r,\vartheta,\varphi) \qquad \text{(III.5)}$$

$$\zeta^0_{,t} = 2\Omega(t,r,\vartheta,\varphi) \qquad \text{(III.6)}$$

$$\zeta^1_{,\vartheta} + \zeta^2_{,r} - 2cotr\zeta^2 = 0 \qquad \text{(III.7)}$$

$$\zeta^2_{,\varphi} + \zeta^3_{,\vartheta} - 2cot\vartheta\zeta^3 = 0 \qquad \text{(III.8)}$$

$$\zeta^1_{,\varphi} + \zeta^3_{,r} - 2cotr\zeta^3 = 0 \qquad \text{(III.9)}$$

$$\zeta^1_{,t} + \zeta^0_{,r} = 0 \qquad \text{(III.10)}$$

$$\zeta^2_{,\mathrm{t}} + \zeta^0_{,\vartheta} = 0 \qquad \text{(III.11)}$$

$$\zeta^3_{,\mathrm{t}} + \zeta^0_{,\varphi} = 0 \qquad \text{(III.12)}$$

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Solve the ten conformal killing's equations as a system with maple see appendix (1) we get:

$$\Omega(t,r,\vartheta,\varphi)=\frac{1}{2}\big((-(c_4 cos\varphi+c_2 sin\varphi)sint+cost(c_3 cos\varphi+c_1 sin\varphi))sin\vartheta+$$
$$cos\vartheta(c_5 cost-c_6 sint))sinr-\frac{1}{2}\ cosr(c_9 sint-c_8\cos t)$$

$$\zeta^0=\Big(\big((c_4 cos\varphi+c_2 sin\varphi)cost+sint(c_3 cos\varphi+c_1 sin\varphi)\big)sin\vartheta+cos\vartheta(c_5 sint+c_6 cost)\Big)sinr+c_8 cosrsint$$
$$+c_9 cosrcost+c_7$$

$$\zeta^1=\Big(\big((-c_4 cos\varphi+c_2 sin\varphi)sint+cost(c_3 cos\varphi+c_1 sin\varphi)\big)cosr+c_{11} sin\varphi+c_{12} cos\varphi)sin\vartheta+cos\vartheta cosr(c_5 cost$$
$$-c_6 sint)+c_9 sinrsint-c_8 sinrcost+c_{10} cos\vartheta\Big)$$

$$\zeta^2=\frac{1}{4}\frac{1}{tanr\ sin\vartheta\ cos\vartheta}((2cos2r-2)tanrcos\vartheta(c_{14} cos\varphi-c_{15} sin\varphi)\sqrt{cos2\vartheta-1}-c_{11}(cos2r-1)cos(\varphi-\vartheta)$$
$$+c_{12}(cos2r-1)sin(\varphi-\vartheta)+\big(c_{11} cos(\varphi+\vartheta)-c_{12} sin(\varphi+\vartheta)\big)cos2r-c_{11} cos(\varphi+\vartheta)$$
$$+c_{12} sin(\varphi+\vartheta)$$
$$-4sin\vartheta sinrtanr((\Big((c_3 cost-c_4 sint)cos\varphi$$
$$+sin\varphi\ \big(cos\varphi(c_1 cost-c_2 sint))sin^2 r+(((c_3 cost-c_4 sint)cosr+c_{12})\big)\ cos\varphi$$
$$+sin\varphi((c_1 cost-c_2 sint)cosr+c_{11}))+cosrsin^2\vartheta$$
$$+cos\vartheta\ \big((c_5 cost-c_6 sint)sin^2 r+\big((c_5 cost-c_6 sint)cosr+c_{10}\big)cosr\big)\ sin\vartheta$$
$$+(c_4 sint-c_3 cost)cos\varphi-sin\varphi(c_1 cost-c_2 sint)\Big))$$

$$\zeta^3=\frac{1}{4tanr}((2-2cos2r)tanr(c_{13} sin\vartheta+cos\vartheta(c_{14} sin\varphi+c_{15} cos\varphi))\sqrt{cos2\vartheta-1}+c_{12}(cos2r-1)\cos(\varphi-\vartheta)$$
$$+c_{11}(cos2r-1)\sin(\varphi-\vartheta)+(-c_{12}\cos(\varphi+\vartheta)-c_{11}\sin(\varphi+\vartheta))cos2r+c_{12}\cos(\varphi+\vartheta)$$
$$+c_{11}\sin(\varphi+\vartheta)+4sin\vartheta tanr\big((c_1 cost-c_2 sint)cos\varphi-(c_3 cost-c_4 sint)sin\varphi\big)sinr)$$

(III.13)

Where $c_i, i=1,2,\ldots$ are integration constants

The conformal killing's vector fields be

$$\zeta=\zeta^0\partial t+\zeta^1\partial r+\zeta^2\partial\vartheta+\zeta^3\partial\varphi$$

Where we replace $\Omega(t,r,\vartheta,\varphi)$ by a constant function we get the homothetic equations which solved by maple see appendix (1)to get: [11],[12]

$\zeta^0=2\psi t+A(\varphi)$, $A(\varphi)$: constant's integration function

$\zeta^1=tanr[kc_1^2 e^{-k\vartheta}(\tan r)^{c_1}(c_3-c_2 e^{2k\vartheta})-2\psi]$, $k=\sqrt{c_1}$

$\zeta^2=(tanr)^{-c_1}e^{-k\vartheta}[c_1^2(\sin r)^2(c_3+c_2 e^{2k\vartheta})]$,

$\zeta^3=\frac{1}{4}B(\varphi,t)(cos2r-1)(cos2\vartheta-1)$, $(\varphi,t\ )$:constant's integration function

(III.14)

Where we replace $\Omega(t,r,\vartheta,\varphi)$ by zero we get the killing's equations which solved with maple see appendix (1) to get:

$\zeta^0=c_1$, $\zeta^1=c_2 cos\vartheta+sin\vartheta(c_3 sin\varphi+c_4 cos\varphi)$

$$\zeta^2=\frac{1}{2}[cos2r(-c_6 cos\varphi-c_5 sin\varphi)+sin\varphi(c_4 sin2r\ cos\vartheta+c_6)-c_2 sin2rsin\vartheta]$$

$$\zeta^3=\frac{1}{4tanrtan\vartheta}\{[c_4 tan\vartheta\cos(\varphi-\vartheta)+c_3 tan\vartheta\sin(\varphi-\vartheta)+tanrcos2\vartheta(c_5 cos\varphi-c_6 sin\varphi+c_7 tan\vartheta)](cos2r-1)\}$$

(III.15)

IV. CONFORMAL KILLING'S EQUATIONS AND THEIR SOLUTION IN LYRA'S GEOMETRY

Where $\Gamma^{\alpha}_{\mu\nu}$ is a Lyra connection form given by (II.2) in equation (II.3) we get the non vanishing forty one connection as: [13],[14]

$$\Gamma^t_{tt} = \phi_t - \frac{1}{2}\phi^t,\ \Gamma^t_{tr} = \Gamma^t_{rt} = \frac{1}{2}\phi_t,\ \ \Gamma^t_{rr} = \frac{1}{2}\phi^t, \Gamma^t_{t\vartheta} = \Gamma^t_{\vartheta t} = \frac{1}{2}\phi_t,$$

$$\Gamma^r_{rr} = \phi_r + \frac{1}{2}\phi^r,\ \Gamma^r_{rt} = \Gamma^r_{tr} = \frac{1}{2}\phi_r, \Gamma^r_{\vartheta\vartheta} = \frac{-\sin r\cos r}{x^o} + \frac{1}{2}\sin^2 r\,\phi^r$$

$$\Gamma^r_{tt} = -\frac{1}{2}\phi^r, \Gamma^r_{\varphi r} = \Gamma^r_{r\varphi} = \frac{1}{2}\phi_r, \Gamma^{\vartheta}_{tt} = -\frac{1}{2}\phi^{\vartheta}, \Gamma^{\vartheta}_{t\vartheta} = \Gamma^{\vartheta}_{\vartheta t} = \frac{1}{2}\phi_{\vartheta}$$

$$\Gamma^t_{\vartheta\vartheta} = \frac{1}{2}\sin^2\vartheta\phi^t, \Gamma^t_{\varphi\varphi} = \frac{1}{2}\sin^2 r\sin^2\vartheta\ \phi^t,\ \Gamma^t_{t\varphi} = \Gamma^t_{\varphi t} = \frac{1}{2}\phi_t$$

$$\Gamma^r_{r\vartheta} = \Gamma^r_{\vartheta r} = \frac{1}{2}\phi_r, \Gamma^r_{\varphi\varphi} = -\frac{\sin r\cos r\sin^2\vartheta}{x^o} + \frac{1}{2}\sin^2 r\sin^2\vartheta\phi^r$$

$$\Gamma^{\varphi}_{rr} = \frac{1}{2}\phi^{\varphi}, \Gamma^{\varphi}_{\vartheta\vartheta} = \frac{1}{2}\sin^2 r\phi^{\varphi}, \Gamma^{\varphi}_{\varphi r} = \Gamma^{\varphi}_{r\varphi} = \frac{\cos r}{\sin r x^o} + \frac{1}{2}\phi_{\varphi}$$

$$\Gamma^{\vartheta}_{\varphi\varphi} = -\frac{\sin\vartheta\cos\vartheta}{x^o} + \frac{1}{2}\sin^2 r\sin^2\vartheta\phi^{\vartheta},\ \Gamma^{\varphi}_{tt} = -\frac{1}{2}\phi^{\varphi}$$

$$\Gamma^{\varphi}_{\varphi\vartheta} = \Gamma^{\varphi}_{\vartheta\varphi} = \frac{\cos\vartheta}{\sin\vartheta x^o} + \frac{1}{2}\phi_{\varphi}, \Gamma^{\vartheta}_{\varphi\vartheta} = \Gamma^{\vartheta}_{\vartheta\varphi} = \frac{1}{2}\phi_{\vartheta}$$

$$\Gamma^{\vartheta}_{r\vartheta} = \Gamma^{\vartheta}_{\vartheta r} = \frac{\cos r}{\sin r x^o} + \frac{1}{2}\phi_{\vartheta},\ \Gamma^{\varphi}_{\vartheta\vartheta} = \frac{1}{2}\sin^2 r\phi^{\varphi}$$

$$\Gamma^{\vartheta}_{rr} = \frac{1}{2}\phi^{\vartheta}, \Gamma^{\vartheta}_{\vartheta\vartheta} = \phi_{\vartheta} + \frac{1}{2}\sin^2 r\phi^{\vartheta}$$

$$\Gamma^{\varphi}_{\varphi\varphi} = \phi_{\varphi} + \frac{1}{2}\sin^2 r\sin^2\vartheta\phi^{\varphi}$$

$$\Gamma^{\varphi}_{\varphi t} = \Gamma^{\varphi}_{t\varphi} = \frac{1}{2}\phi_{\varphi} \tag{IV.1}$$

From equation (II.4) we get:

$$\phi^0 = \beta\quad, \phi^1 = -\beta, \phi^2 = -\frac{\beta}{\sin^2 r}, \phi^3 = -\frac{\beta}{\sin^2 r\sin^2\vartheta} \tag{IV.2}$$

We use equation (II.5) to deduce the following system of ten Conformal killing's equations in Lyra Geometry :

$$\zeta^3_{,\varphi} + (\frac{1}{2}\beta)\zeta^0 + (\frac{\cos r}{\sin r x^o})\zeta^1 + \left(\frac{\cos\vartheta}{\sin\vartheta x^o}\right)\zeta^2 + \left(\frac{1}{2}\sin^2 r\sin^2\vartheta\right)\zeta^3 = \Omega(t,r,\vartheta,\varphi) \tag{IV.3}$$

$$\zeta^2_{,\vartheta} + (\frac{\beta}{2})\zeta^0 + \left(\frac{\cos r}{\sin r x^o}\right)\zeta^1 = \Omega(t,r,\vartheta,\varphi) \tag{IV.4}$$

$$\zeta^0_{,t} + \frac{1}{2}\beta\zeta^0 = \Omega(t,r,\vartheta,\varphi) \tag{IV.5}$$

$$\zeta^1_{,r} + (\frac{1}{2}\beta)\zeta^0 = \Omega(t,r,\vartheta,\varphi) \tag{IV.6}$$

$$\sin^2 r\sin^2\vartheta\zeta^3_{,t} - \zeta^0_{,\varphi} = 0 \tag{IV.7}$$

$$\sin^2 r\sin^2\vartheta\zeta^3_{,r} + \zeta^1_{,\varphi} = 0 \tag{IV.8}$$

$$\sin^2 r\zeta^2_{,t} - \zeta^0_{,\vartheta} = 0 \tag{IV.9}$$

$$\sin^2 r\zeta^2_{,r} + \zeta^1_{,\vartheta} = 0 \tag{IV.10}$$

$$\sin^2\vartheta\zeta^3_{,\vartheta} + \zeta^2_{,\varphi} = 0 \tag{IV.12}$$

$$\zeta^0_{,r} - \zeta^1_{,t} = 0 \tag{IV.13}$$

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Solve the ten conformal killing's equations as a system with maple see appendix (2)we get:

$$\Omega(t,r,\vartheta,\varphi) = \frac{\beta}{2}c_1\,,\zeta^0 = c_1,\ \zeta^1 = 0, \zeta^2 = \sqrt{x^o}\left[c_4 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) - c_3 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right],$$

$$\zeta^3 = c_2 + cotan\vartheta\left[c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + c_4 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right] \qquad \text{(IV.14)}$$

The conformal killing's vector fields in Lyra Geometry are:

$$\zeta = c_1\partial t + 0\partial r + \sqrt{x^o}\left[c_4 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) - c_3 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\partial\vartheta + \left\{c_2 + cotan\vartheta\left[c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + c_4 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\right\}\partial\varphi \qquad \text{(IV.15)}$$

The solution of the homothetic's equations by maple see appendix (2) is:

$$\zeta^0 = \frac{2\psi}{\beta},\ \zeta^1 = 0, \zeta^2 = \sqrt{x^o}\left[c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) - c_2 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right],$$

$$\zeta^3 = \left\{c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + cotan(\vartheta)\left[c_2 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + c_3 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\right\}$$

The homothetic vector fields are:

$$\zeta = \frac{2\psi}{\beta}\partial t + 0\partial r + \sqrt{x^o}\left[c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) - c_2 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\partial\vartheta + \left\{c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + cotan(\vartheta)\left[c_2 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + c_3 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\right\}\partial\varphi \qquad \text{(IV.16)}$$

The solution of the killing's equations by maple see appendix (2) are:

$$\zeta^0 = 0,\ \zeta^1 = 0, \zeta^2 = \sqrt{x^o}\left[c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) - c_2 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right],$$

$$\zeta^3 = -\left\{c_1 + cotan(\vartheta)\left[c_2 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + c_3 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\right\}$$

The killing vector fields are:

$$\zeta = 0\partial t + 0\partial r + \sqrt{x^o}\left[c_3 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) - c_2 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\partial\vartheta - \left\{c_1 + cotan(\vartheta)\left[c_2 sin\left(\frac{\varphi}{\sqrt{x^o}}\right) + c_3 cos\left(\frac{\varphi}{\sqrt{x^o}}\right)\right]\right\}\partial\varphi \qquad \text{(IV.17)}$$

Particular case: where $c_i = x^o = 1$, the conformal, homothetic and killing vector fields are respectively:

$$\zeta = \partial t + 0\partial r + [sin(\varphi) - cos(\varphi)]\partial\vartheta + \{1 + cotan(\vartheta)[sin(\varphi) + cos(\varphi)]\}\partial\varphi$$

$$\zeta = \frac{2\psi}{\beta}\partial t + 0\partial r + [sin(\varphi) - cos(\varphi)]\partial\vartheta + \{sin(\varphi) + cotan(\vartheta)[sin(\varphi) + cos(\varphi)]\}\partial\varphi$$

$$\zeta = 0\partial t + 0\partial r + [sin(\varphi) - cos(\varphi)]\partial\vartheta - \{1 + cotan(\vartheta)[sin(\varphi) + cos(\varphi)]\}\partial\varphi \qquad \text{(IV.18)}$$

V. DISCUSSION

In this paper we get the ten conformal killing's equations of Einstein Closed Static Universe metric in Riemann geometry and in Lyra geometry by ordinary method and by Maple17and solve it by Maple17 program and get the conformal vector field in the Riemann geometry and in Lyra geometry, as a special case in Lyra geometry where $c_i = x^o = 1$ and getting the vectors .

VI. CONCLUSION

The components of conformal vector field in Riemannian geometry are so complicated than it in Lyra Geometry also the homothetic and killing vector fields since Lyra connections has the essential role to Simplify equations and simplify their solution. Although the solutions of equations appear to be long, but if these solutions are compensated in any of their equations, they will be achieved see Appendix(3). The solution may be simplified under specific conditions and specific values of the given variables.

VII. RECOMMENDATIONS

It can be study the same equations for Closed Static Universe metric in teleparallel gravity and using maple to get conformal equations and its solution for more difficult models

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APPENDIX - 1

> *with*(*DifferentialGeometry*) : *with*(*Tensor*) :

"Closed Static Universe)" "Riemannian geometry "

> *DGsetup*([*t*, r, ϑ, φ], *M*)

frame name: M

M > $g := evalDG(\mathrm{dt} \,\&t\, \mathrm{dt} - \mathrm{dr} \,\&t\, \mathrm{dr} - \sin^2(r)\cdot(\mathrm{d}\vartheta \,\&t\, \mathrm{d}\vartheta + \sin^2(\vartheta)\mathrm{d}\varphi\&t\,\mathrm{d}\varphi))$

$g := dt\, dt - dr\, dr - \sin(r)^2\, d\vartheta\, d\vartheta - \sin(r)^2 \sin(\vartheta)^2\, d\varphi\, d\varphi$

> $H := InverseMetric(g)$

$$H := D\_t\, D\_t - D\_r\, D\_r - \frac{1}{\sin(r)^2} D\_\vartheta\, D\_\vartheta - \frac{1}{\sin(r)^2 \sin(\vartheta)^2} D\_\varphi\, D\_\varphi$$

M > $C1 := Christoffel(g, \text{"FirstKind"})$

$$\begin{aligned} C1 :=\; & \sin(r)\cos(r)\, dr\, d\vartheta\, d\vartheta + \sin(r)\sin(\vartheta)^2\cos(r)\, dr\, d\varphi\, d\varphi - \sin(r)\cos(r)\, d\vartheta\, dr\, d\vartheta \\ & - \sin(r)\cos(r)\, d\vartheta\, d\vartheta\, dr + \sin(r)^2\sin(\vartheta)\cos(\vartheta)\, d\vartheta\, d\varphi\, d\varphi - \sin(r)\sin(\vartheta)^2\cos(r)\, d\varphi\, dr\, d\varphi \\ & - \sin(r)^2\sin(\vartheta)\cos(\vartheta)\, d\varphi\, d\vartheta\, d\varphi - \sin(r)\sin(\vartheta)^2\cos(r)\, d\varphi\, d\varphi\, dr \\ & - \sin(r)^2\sin(\vartheta)\cos(\vartheta)\, d\varphi\, d\varphi\, d\vartheta \end{aligned}$$

M > $C2 := Christoffel(g, \text{"SecondKind"})$

$$\begin{aligned} C2 :=\; & -\sin(r)\cos(r)\, D\_r\, d\vartheta\, d\vartheta - \sin(r)\sin(\vartheta)^2\cos(r)\, D\_r\, d\varphi\, d\varphi + \frac{\cos(r)}{\sin(r)} D\_\vartheta\, dr\, d\vartheta \\ & + \frac{\cos(r)}{\sin(r)} D\_\vartheta\, d\vartheta\, dr - \sin(\vartheta)\cos(\vartheta)\, D\_\vartheta\, d\varphi\, d\varphi + \frac{\cos(r)}{\sin(r)} D\_\varphi\, dr\, d\varphi + \frac{\cos(\vartheta)}{\sin(\vartheta)} D\_\varphi\, d\vartheta\, d\varphi \\ & + \frac{\cos(r)}{\sin(r)} D\_\varphi\, d\varphi\, dr + \frac{\cos(\vartheta)}{\sin(\vartheta)} D\_\varphi\, d\varphi\, d\vartheta \end{aligned}$$

M > for _*eq* in *sys1* do _*eq* end do;

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial r}\_F3(t, r, \vartheta, \varphi)\right)\sin(r) - 2\cos(r)\_F3(t, r, \vartheta, \varphi) + \left(\frac{\partial}{\partial \vartheta}\_F2(t, r, \vartheta, \varphi)\right)\sin(r)}{\sin(r)} = 0$$

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial \varphi}\_F3(t, r, \vartheta, \varphi)\right)\sin(\vartheta) + \left(\frac{\partial}{\partial \vartheta}\_F4(t, r, \vartheta, \varphi)\right)\sin(\vartheta) - 2\cos(\vartheta)\_F4(t, r, \vartheta, \varphi)}{\sin(\vartheta)} = 0$$

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial r}\_F4(t, r, \vartheta, \varphi)\right)\sin(r) - 2\cos(r)\_F4(t, r, \vartheta, \varphi) + \left(\frac{\partial}{\partial \varphi}\_F2(t, r, \vartheta, \varphi)\right)\sin(r)}{\sin(r)} = 0$$

$$\sin(r)\cos(r)\_F2(t, r, \vartheta, \varphi) + \frac{\partial}{\partial \vartheta}\_F3(t, r, \vartheta, \varphi) = -2\sin(r)^2\,\Omega(t, r, \vartheta, \varphi)$$

$$\frac{1}{2}\frac{\partial}{\partial t}\_F2(t, r, \vartheta, \varphi) + \frac{1}{2}\frac{\partial}{\partial r}\_F1(t, r, \vartheta, \varphi) = 0$$

$$\frac{1}{2}\frac{\partial}{\partial t}\_F3(t, r, \vartheta, \varphi) + \frac{1}{2}\frac{\partial}{\partial \vartheta}\_F1(t, r, \vartheta, \varphi) = 0$$

$$\frac{1}{2}\frac{\partial}{\partial t}\_F4(t, r, \vartheta, \varphi) + \frac{1}{2}\frac{\partial}{\partial \varphi}\_F1(t, r, \vartheta, \varphi) = 0$$

$$\begin{aligned} & \sin(\vartheta)\cos(\vartheta)\_F3(t, r, \vartheta, \varphi) - \sin(r)\cos(r)\cos(\vartheta)^2\_F2(t, r, \vartheta, \varphi) + \sin(r)\cos(r)\_F2(t, r, \vartheta, \varphi) \\ & + \frac{\partial}{\partial \varphi}\_F4(t, r, \vartheta, \varphi) = -2\sin(r)^2\sin(\vartheta)^2\,\Omega(t, r, \vartheta, \varphi) \end{aligned}$$

$$\frac{\partial}{\partial t}\_F1(t, r, \vartheta, \varphi) = 2\,\Omega(t, r, \vartheta, \varphi)$$

$$\frac{\partial}{\partial r}\_F2(t,r,\vartheta,\varphi) = -2\,\Omega(t,r,\vartheta,\varphi)$$

M $>pdsolve(sys1)$

$$\Big\{\Omega(t,r,\vartheta,\varphi) = \frac{1}{2}\Big(\big(-(\cos(\varphi)\,\_C4+\sin(\varphi)\,\_C2)\sin(t)+\cos(t)\,(\cos(\varphi)\,\_C3 + \sin(\varphi)\,\_C1)\big)\sin(\vartheta)+\cos(\vartheta)\,(\_C5\cos(t)-\_C6\sin(t))\Big)\sin(r)-\frac{1}{2}\cos(r)\,(\_C9\sin(t) - \_C8\cos(t)),\ \_F1(t,r,\vartheta,\varphi) = \Big(\big((\cos(\varphi)\,\_C4+\sin(\varphi)\,\_C2)\cos(t)+\sin(t)\,(\cos(\varphi)\,\_C3 + \sin(\varphi)\,\_C1)\big)\sin(\vartheta)+\cos(\vartheta)\,(\_C5\sin(t)+\_C6\cos(t))\Big)\sin(r)+\sin(t)\cos(r)\,\_C8 + \cos(t)\cos(r)\,\_C9+\_C7,\ \_F2(t,r,\vartheta,\varphi) = \Big(\big(-(\cos(\varphi)\,\_C4+\sin(\varphi)\,\_C2)\sin(t) + \cos(t)\,(\cos(\varphi)\,\_C3+\sin(\varphi)\,\_C1)\big)\cos(r)+\_C11\sin(\varphi)+\_C12\cos(\varphi)\Big)\sin(\vartheta) + \cos(\vartheta)\,(\_C5\cos(t)-\_C6\sin(t))\cos(r)+\sin(t)\sin(r)\,\_C9-\cos(t)\sin(r)\,\_C8 + \_C10\cos(\vartheta),\ \_F3(t,r,\vartheta,\varphi) = \frac{1}{4}\,\frac{1}{\tan(r)\sin(\vartheta)\cos(\vartheta)}\Big(2\tan(r)\cos(\vartheta)\,(\_C14\cos(\varphi) - \_C15\sin(\varphi))\,(-1+\cos(2r))\sqrt{-1+\cos(2\vartheta)}-\_C11\,(-1+\cos(2r))\cos(\varphi-\vartheta)+\_C12\,(-1+\cos(2r))\sin(\varphi-\vartheta)+(\_C11\cos(\varphi+\vartheta)-\_C12\sin(\varphi+\vartheta))\cos(2r)-\_C11\cos(\varphi+\vartheta) + \_C12\sin(\varphi+\vartheta)-4\sin(r)\Big(\Big(\big(((\_C3\cos(t)-\_C4\sin(t))\cos(\varphi)+\sin(\varphi)\,(\cos(t)\,\_C1 - \_C2\sin(t)))\sin(r)^2+(((\_C3\cos(t)-\_C4\sin(t))\cos(r)+\_C12)\cos(\varphi) + \sin(\varphi)\,((\cos(t)\,\_C1-\_C2\sin(t))\cos(r)+\_C11))\cos(r)\big)\sin(\vartheta)^2+\big((\_C5\cos(t) - \_C6\sin(t))\sin(r)^2+((\_C5\cos(t)-\_C6\sin(t))\cos(r)+\_C10)\cos(r)\big)\cos(\vartheta)\sin(\vartheta)+(-\_C3\cos(t)+\_C4\sin(t))\cos(\varphi)-\sin(\varphi)\,(\cos(t)\,\_C1-\_C2\sin(t))\Big)\sin(\vartheta)\tan(r)\Big)\Big),\ \_F4(t,r, \vartheta,\varphi) = \frac{1}{4}\,\frac{1}{\tan(r)}\Big(-2\tan(r)\,(\_C13\sin(\vartheta)+\cos(\vartheta)\,(\_C14\sin(\varphi)+\_C15\cos(\varphi)))\,(-1 + \cos(2r))\sqrt{-1+\cos(2\vartheta)}+\_C12\,(-1+\cos(2r))\cos(\varphi-\vartheta)+\_C11\,(-1+\cos(2r))\sin(\varphi - \vartheta)+(-\_C12\cos(\varphi+\vartheta)-\_C11\sin(\varphi+\vartheta))\cos(2r)+\_C12\cos(\varphi+\vartheta)+\_C11\sin(\varphi + \vartheta)+4\,((\cos(t)\,\_C1-\_C2\sin(t))\cos(\varphi)-(\_C3\cos(t) - \_C4\sin(t))\sin(\varphi))\tan(r)\sin(\vartheta)\sin(r)\Big)\Big\}$$

M > for _eq in *sys2* do _eq end do; "Homothetic equations in *Riemannian geometry*"

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial\vartheta}\_F2(t,r,\vartheta,\varphi)\right)\sin(r)+\left(\frac{\partial}{\partial r}\_F3(t,r,\vartheta,\varphi)\right)\sin(r)-2\cos(r)\,\_F3(t,r,\vartheta,\varphi)}{\sin(r)} = 0$$

$$\frac{1}{2}\,\frac{-2\cos(\vartheta)\,\_F4(t,r,\vartheta,\varphi)+\left(\frac{\partial}{\partial\vartheta}\_F4(t,r,\vartheta,\varphi)\right)\sin(\vartheta)+\left(\frac{\partial}{\partial\varphi}\_F3(t,r,\vartheta,\varphi)\right)\sin(\vartheta)}{\sin(\vartheta)}=0$$

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial\varphi}\_F2(t,r,\vartheta,\varphi)\right)\sin(r)+\left(\frac{\partial}{\partial r}\_F4(t,r,\vartheta,\varphi)\right)\sin(r)-2\cos(r)\,\_F4(t,r,\vartheta,\varphi)}{\sin(r)}=0$$

$$\cos(r)\sin(r)\,\_F2(t,r,\vartheta,\varphi)+\frac{\partial}{\partial\vartheta}\_F3(t,r,\vartheta,\varphi)=-2\,\psi\sin(r)^2$$

$$\frac{1}{2}\,\frac{\partial}{\partial t}\_F2(t,r,\vartheta,\varphi)+\frac{1}{2}\,\frac{\partial}{\partial r}\_F1(t,r,\vartheta,\varphi)=0$$

$$\frac{1}{2}\,\frac{\partial}{\partial t}\_F3(t,r,\vartheta,\varphi)+\frac{1}{2}\,\frac{\partial}{\partial\vartheta}\_F1(t,r,\vartheta,\varphi)=0$$

$$\frac{1}{2}\,\frac{\partial}{\partial t}\_F4(t,r,\vartheta,\varphi)+\frac{1}{2}\,\frac{\partial}{\partial\varphi}\_F1(t,r,\vartheta,\varphi)=0$$

$$\sin(\vartheta)\cos(\vartheta)\,\_F3(t,r,\vartheta,\varphi)-\cos(\vartheta)^2\,\_F2(t,r,\vartheta,\varphi)\sin(r)\cos(r)+\cos(r)\sin(r)\,\_F2(t,r,\vartheta,\varphi)+\frac{\partial}{\partial\varphi}\_F4(t,r,\vartheta,\varphi)=-2\,\psi\sin(r)^2\sin(\vartheta)^2$$

$$\frac{\partial}{\partial t}\_F1(t,r,\vartheta,\varphi)=2\,\psi$$

$$\frac{\partial}{\partial r}\_F2(t,r,\vartheta,\varphi)=-2\,\psi$$

pdsolve([*e1*, *e2*, *e3*, *e4*, *e5*, *e6*, *e9*])

$$\Bigg\{\_F1(t,r,\vartheta,\varphi)=2\,\psi\,t+\_F5(\varphi),\ \_F2(t,r,\vartheta,\varphi)=\frac{1}{\cos(r)\sin(r)\,e^{\sqrt{-\_c_1}\,\vartheta}}\Bigg(\sqrt{\_c_1\,\_C1^2\,(\cos(r)-1)\,(\cos(r)+1)}\left(\_C2\left(e^{\sqrt{-\_c_1}\,\vartheta}\right)^2-\_C3\right)\left(\frac{\sin(r)}{\cos(r)}\right)^{-\_c_1}-2\,\psi\sin(r)^2\,e^{\sqrt{-\_c_1}\,\vartheta}\Bigg),\ \_F3(t,r,\vartheta,\varphi)=-\frac{\left(\frac{\sin(r)}{\cos(r)}\right)^{-\_c_1}\_C1^2\,(\cos(r)-1)\,(\cos(r)+1)\left(\_C2\left(e^{\sqrt{-\_c_1}\,\vartheta}\right)^2+\_C3\right)}{e^{\sqrt{-\_c_1}\,\vartheta}},\ \_F4(t,r,\vartheta,\varphi)=\frac{1}{4}\,\_F6(t,\varphi)\,(-1+\cos(2\,\vartheta))\,(-1+\cos(2\,r))\Bigg\}$$

M > for _*eq* in sys3 do _*eq* end do; *Killing equations* in *Riemannian geometry"*

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial\vartheta}\_F2(t,r,\vartheta,\varphi)\right)\sin(r)+\left(\frac{\partial}{\partial r}\_F3(t,r,\vartheta,\varphi)\right)\sin(r)-2\cos(r)\,\_F3(t,r,\vartheta,\varphi)}{\sin(r)}=0$$

$$\frac{1}{2}\,\frac{-2\cos(\vartheta)\,\_F4(t,r,\vartheta,\varphi)+\left(\frac{\partial}{\partial\vartheta}\_F4(t,r,\vartheta,\varphi)\right)\sin(\vartheta)+\left(\frac{\partial}{\partial\varphi}\_F3(t,r,\vartheta,\varphi)\right)\sin(\vartheta)}{\sin(\vartheta)}=0$$

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial\varphi}\_F2(t,r,\vartheta,\varphi)\right)\sin(r)+\left(\frac{\partial}{\partial r}\_F4(t,r,\vartheta,\varphi)\right)\sin(r)-2\cos(r)\,\_F4(t,r,\vartheta,\varphi)}{\sin(r)}=0$$

$$\cos(r)\sin(r)\,\_F2(t,r,\vartheta,\varphi)+\frac{\partial}{\partial\vartheta}\_F3(t,r,\vartheta,\varphi)=0$$

$$\frac{1}{2}\frac{\partial}{\partial t}\_F2(t,r,\vartheta,\varphi)+\frac{1}{2}\frac{\partial}{\partial r}\_F1(t,r,\vartheta,\varphi)=0$$

$$\frac{1}{2}\frac{\partial}{\partial t}\_F3(t,r,\vartheta,\varphi)+\frac{1}{2}\frac{\partial}{\partial\vartheta}\_F1(t,r,\vartheta,\varphi)=0$$

$$\frac{1}{2}\frac{\partial}{\partial t}\_F4(t,r,\vartheta,\varphi)+\frac{1}{2}\frac{\partial}{\partial\varphi}\_F1(t,r,\vartheta,\varphi)=0$$

$$\sin(\vartheta)\cos(\vartheta)\,\_F3(t,r,\vartheta,\varphi)-\cos(\vartheta)^2\,\_F2(t,r,\vartheta,\varphi)\sin(r)\cos(r)+\cos(r)\sin(r)\,\_F2(t,r,\vartheta,\varphi)$$
$$+\frac{\partial}{\partial\varphi}\_F4(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial t}\_F1(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial r}\_F2(t,r,\vartheta,\varphi)=0$$

> *pdsolve*(*sys3*)

$$\{\_F1(t,r,\vartheta,\varphi)=\_C1,\ \_F2(t,r,\vartheta,\varphi)=(\_C3\sin(\varphi)+\cos(\varphi)\,\_C4)\sin(\vartheta)+\_C2\cos(\vartheta),\ \_F3(t,r,\vartheta,$$
$$\varphi)=\frac{1}{2}(-\_C6\cos(\varphi)-\_C5\sin(\varphi))\cos(2r)+\frac{1}{2}(2\cos(\vartheta)\,\_C4\cos(r)\sin(r)+\_C6)\cos(\varphi)$$
$$+\frac{1}{2}(2\cos(\vartheta)\,\_C3\cos(r)\sin(r)+\_C5)\sin(\varphi)-\sin(r)\cos(r)\sin(\vartheta)\,\_C2,\ \_F4(t,r,\vartheta,\varphi)$$
$$=\frac{1}{4}\frac{1}{\tan(r)\tan(\vartheta)}((\cos(\varphi-\vartheta)\tan(\vartheta)\,\_C4+\sin(\varphi-\vartheta)\tan(\vartheta)\,\_C3+\tan(r)(\_C5\cos(\varphi)$$
$$-\_C6\sin(\varphi)+\_C7\tan(\vartheta))\cos(2\vartheta)-\cos(\varphi+\vartheta)\tan(\vartheta)\,\_C4-\sin(\varphi+\vartheta)\tan(\vartheta)\,\_C3$$
$$-\tan(r)(\_C5\cos(\varphi)-\_C6\sin(\varphi)+\_C7\tan(\vartheta)))(-1+\cos(2r)))\}$$

APPENDIX - 2

M > for _*eq* in *sys1* do _*eq* end do; *lyra*

$$\left(\frac{\partial}{\partial r}\_F3(t,r,\vartheta,\varphi)\right)\sin(r)^2+\frac{\partial}{\partial\vartheta}\_F2(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial\varphi}\_F3(t,r,\vartheta,\varphi)+\left(\frac{\partial}{\partial\vartheta}\_F4(t,r,\vartheta,\varphi)\right)\sin(\vartheta)^2=0$$

$$\left(\frac{\partial}{\partial r}\_F4(t,r,\vartheta,\varphi)\right)\sin(r)^2\sin(\vartheta)^2+\frac{\partial}{\partial\varphi}\_F2(t,r,\vartheta,\varphi)=0$$

$$\frac{\cos(r)\,\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{\partial}{\partial\vartheta}\_F3(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\,\_F1(t,r,\vartheta,\varphi)=\Omega(t,r,\vartheta,\varphi)$$

$$\frac{\partial}{\partial r}\_F1(t,r,\vartheta,\varphi)-\left(\frac{\partial}{\partial t}\_F2(t,r,\vartheta,\varphi)\right)=0$$

$$\sin(r)^2\left(\frac{\partial}{\partial t}\_F3(t,r,\vartheta,\varphi)\right)-\left(\frac{\partial}{\partial\vartheta}\_F1(t,r,\vartheta,\varphi)\right)=0$$

$$\sin(r)^2\sin(\vartheta)^2\left(\frac{\partial}{\partial t}\_F4(t,r,\vartheta,\varphi)\right)-\left(\frac{\partial}{\partial\varphi}\_F1(t,r,\vartheta,\varphi)\right)=0$$

$$\frac{\cos(\vartheta)\,\_F3(t,r,\vartheta,\varphi)}{a\sin(\vartheta)}+\frac{\cos(r)\,\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{1}{2}\beta\,\_F1(t,r,\vartheta,\varphi)+\frac{\partial}{\partial\varphi}\_F4(t,r,\vartheta,\varphi)$$
$$=\Omega(t,r,\vartheta,\varphi)$$

$$\frac{\partial}{\partial t}\_F1(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=\Omega(t,r,\vartheta,\varphi)$$

$$\frac{\partial}{\partial r}\_F2(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=\Omega(t,r,\vartheta,\varphi)$$

M > *pdsolve*(*sys1*)

$$\left\{\Omega(t,r,\vartheta,\varphi)=\frac{1}{2}\beta\_C1, \_F1(t,r,\vartheta,\varphi)=\_C1, \_F2(t,r,\vartheta,\varphi)=0, \_F3(t,r,\vartheta,\varphi)=\left(\_C4\sin\left(\frac{\varphi}{\sqrt{a}}\right)-\_C3\cos\left(\frac{\varphi}{\sqrt{a}}\right)\right)\sqrt{a}, \_F4(t,r,\vartheta,\varphi)=\_C2+\frac{\cos(\vartheta)\left(\_C3\sin\left(\frac{\varphi}{\sqrt{a}}\right)+\_C4\cos\left(\frac{\varphi}{\sqrt{a}}\right)\right)}{\sin(\vartheta)}\right\}$$

M > for_*eq* in *sys2* do_*eq* end do; "Homothetic equations in Lyra Geometry"

$$\left(\frac{\partial}{\partial r}\_F3(t,r,\vartheta,\varphi)\right)\sin(r)^2+\frac{\partial}{\partial\vartheta}\_F2(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial\varphi}\_F3(t,r,\vartheta,\varphi)+\left(\frac{\partial}{\partial\vartheta}\_F4(t,r,\vartheta,\varphi)\right)\sin(\vartheta)^2=0$$

$$\left(\frac{\partial}{\partial r}\_F4(t,r,\vartheta,\varphi)\right)\sin(r)^2\sin(\vartheta)^2+\frac{\partial}{\partial\varphi}\_F2(t,r,\vartheta,\varphi)=0$$

$$\frac{\cos(r)\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{\partial}{\partial\vartheta}\_F3(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=\psi$$

$$\frac{\partial}{\partial r}\_F1(t,r,\vartheta,\varphi)-\left(\frac{\partial}{\partial t}\_F2(t,r,\vartheta,\varphi)\right)=0$$

$$\sin(r)^2\left(\frac{\partial}{\partial t}\_F3(t,r,\vartheta,\varphi)\right)-\left(\frac{\partial}{\partial\vartheta}\_F1(t,r,\vartheta,\varphi)\right)=0$$

$$\sin(r)^2\sin(\vartheta)^2\left(\frac{\partial}{\partial t}\_F4(t,r,\vartheta,\varphi)\right)-\left(\frac{\partial}{\partial\varphi}\_F1(t,r,\vartheta,\varphi)\right)=0$$

$$\frac{\cos(\vartheta)\_F3(t,r,\vartheta,\varphi)}{a\sin(\vartheta)}+\frac{\cos(r)\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)+\frac{\partial}{\partial\varphi}\_F4(t,r,\vartheta,\varphi)=\psi$$

$$\frac{\partial}{\partial t}\_F1(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=\psi$$

$$\frac{\partial}{\partial r}\_F2(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=\psi$$

M > *pdsolve*(*sys2*)

$$\left\{\_F1(t,r,\vartheta,\varphi)=\frac{2\psi}{\beta}, \_F2(t,r,\vartheta,\varphi)=0, \_F3(t,r,\vartheta,\varphi)=\left(\_C3\sin\left(\frac{\varphi}{\sqrt{a}}\right)-\_C2\cos\left(\frac{\varphi}{\sqrt{a}}\right)\right)\sqrt{a}, \_F4(t,r,\vartheta,\varphi)=\_C1+\frac{\cos(\vartheta)\left(\_C2\sin\left(\frac{\varphi}{\sqrt{a}}\right)+\_C3\cos\left(\frac{\varphi}{\sqrt{a}}\right)\right)}{\sin(\vartheta)}\right\}$$

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M > for_eq in sys3 do_eq end do;

$$\left(\frac{\partial}{\partial r}\_F3(t,r,\vartheta,\varphi)\right)\sin(r)^2+\frac{\partial}{\partial\vartheta}\_F2(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial\varphi}\_F3(t,r,\vartheta,\varphi)+\left(\frac{\partial}{\partial\vartheta}\_F4(t,r,\vartheta,\varphi)\right)\sin(\vartheta)^2=0$$

$$\left(\frac{\partial}{\partial r}\_F4(t,r,\vartheta,\varphi)\right)\sin(r)^2\sin(\vartheta)^2+\frac{\partial}{\partial\varphi}\_F2(t,r,\vartheta,\varphi)=0$$

$$\frac{\cos(r)\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{\partial}{\partial\vartheta}\_F3(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial r}\_F1(t,r,\vartheta,\varphi)-\left(\frac{\partial}{\partial t}\_F2(t,r,\vartheta,\varphi)\right)=0$$

$$\sin(r)^2\left(\frac{\partial}{\partial t}\_F3(t,r,\vartheta,\varphi)\right)-\left(\frac{\partial}{\partial\vartheta}\_F1(t,r,\vartheta,\varphi)\right)=0$$

$$\sin(r)^2\sin(\vartheta)^2\left(\frac{\partial}{\partial t}\_F4(t,r,\vartheta,\varphi)\right)-\left(\frac{\partial}{\partial\varphi}\_F1(t,r,\vartheta,\varphi)\right)=0$$

$$\frac{\cos(\vartheta)\_F3(t,r,\vartheta,\varphi)}{a\sin(\vartheta)}+\frac{\cos(r)\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)+\frac{\partial}{\partial\varphi}\_F4(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial t}\_F1(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=0$$

$$\frac{\partial}{\partial r}\_F2(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\_F1(t,r,\vartheta,\varphi)=0$$

M > *pdsolve*(*sys3*)

$$\left\{\_F1(t,r,\vartheta,\varphi)=0,\_F2(t,r,\vartheta,\varphi)=0,\_F3(t,r,\vartheta,\varphi)=-\left(\_C2\cos\left(\frac{\varphi}{\sqrt{a}}\right)-\_C3\sin\left(\frac{\varphi}{\sqrt{a}}\right)\right)\sqrt{a},\_F4(t,r,\vartheta,\varphi)=\_C1+\frac{\cos(\vartheta)\left(\_C2\sin\left(\frac{\varphi}{\sqrt{a}}\right)+\_C3\cos\left(\frac{\varphi}{\sqrt{a}}\right)\right)}{\sin(\vartheta)}\right\}$$

APPENDIX - 3

M > **"Conformal equations in Reminnian Geometry"**

M >

$$L.H.S\,III.5=\frac{\partial}{\partial r}\_F2(t,r,\vartheta,\varphi):=\frac{\partial}{\partial r}((-(\cos(\varphi)\_C4+\sin(\varphi)\_C2)\sin(t)+\cos(t)(\cos(\varphi)\_C3+\sin(\varphi)\_C1))\cos(r)+\_C11\sin(\varphi)+\_C12\cos(\varphi))\sin(\vartheta)+\cos(\vartheta)(\_C5\cos(t)-\_C6\sin(t))\cos(r)+\sin(r)\sin(t)\_C9-\sin(r)\cos(t)\_C8+\_C10\cos(\vartheta)$$

$$-(-(\cos(\varphi)\_C4+\sin(\varphi)\_C2)\sin(t)+\cos(t)(\cos(\varphi)\_C3+\sin(\varphi)\_C1))\sin(r)\sin(\vartheta)+\cos(\vartheta)(\_C5\cos(t)-\_C6\sin(t))\cos(r)+\sin(r)\sin(t)\_C9-\sin(r)\cos(t)\_C8+\_C10\cos(\vartheta)$$

M > $R.H.S{=}-2\,\Omega(t,r,\vartheta,\varphi) := -2\cdot\frac{1}{2}\big(\big(-(\cos(\varphi)\,\_C4+\sin(\varphi)\,\_C2)\sin(t)+\cos(t)\,(\cos(\varphi)\,\_C3$
$+\sin(\varphi)\,\_C1)\big)\sin(\vartheta)+\cos(\vartheta)\,(\_C5\cos(t)-\_C6\sin(t))\big)\sin(r)-\frac{1}{2}\cos(r)\,(\_C9\sin(t)$
$-\_C8\cos(t))$

$$-\big(\big(-(\cos(\varphi)\,\_C4+\sin(\varphi)\,\_C2)\sin(t)+\cos(t)\,(\cos(\varphi)\,\_C3+\sin(\varphi)\,\_C1)\big)\sin(\vartheta)$$
$$+\cos(\vartheta)\,(\_C5\cos(t)-\_C6\sin(t))\big)\sin(r)-\frac{1}{2}\cos(r)\,(\_C9\sin(t)-\_C8\cos(t))$$

M > "Conformal equations in Lyra Geometry"

M > $R.H.S\,IV10 := \left(\frac{\partial}{\partial r}\_F3(t,r,\vartheta,\varphi)\right)\sin(r)^2+\frac{\partial}{\partial\vartheta}\_F2(t,r,\vartheta,\varphi)$

$$IV10 := \left(\frac{\partial}{\partial r}\_F3(t,r,\vartheta,\varphi)\right)\sin(r)^2+\frac{\partial}{\partial\vartheta}\_F2(t,r,\vartheta,\varphi)$$

M > $R.H.S\,IV10 := \sin(r)^2\frac{d}{dr}(0)+\frac{d}{d\vartheta}\_C1$

M > $lL.H.S := 0$

M > $R.H.SIV3 := \frac{\cos(r)\,\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{\partial}{\partial\vartheta}\_F3(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\,\_F1(t,r,\vartheta,\varphi)$

$$IV3 := \frac{\cos(r)\,\_F2(t,r,\vartheta,\varphi)}{a\sin(r)}+\frac{\partial}{\partial\vartheta}\_F3(t,r,\vartheta,\varphi)+\frac{1}{2}\beta\,\_F1(t,r,\vartheta,\varphi)=\Omega(t,r,\vartheta,\varphi)$$

M >

M > $R.HSIV3 := \frac{\cos(r)\cdot 0}{a\sin(r)}+\frac{\partial}{\partial\vartheta}\left(\_C4\sin\left(\frac{\varphi}{\sqrt{a}}\right)-\_C3\cos\left(\frac{\varphi}{\sqrt{a}}\right)\right)+\frac{1}{2}\beta\,\_C1$

$$\frac{1}{2}\beta\,\_C1$$

M > $R.H.S := \Omega(t,r,\vartheta,\varphi)=\frac{1}{2}\beta\,\_C1$

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